

6.1

Prepping for the Robot Challenge

Solving Linear Systems Graphically and Algebraically

LEARNING GOALS

In this lesson, you will:

- Write systems of linear equations.
- Graph systems of linear equations.
- Determine the intersection point, or break-even point, from a graph.
- Use the substitution method to determine the intersection point.
- Understand that systems of equations can have one, zero, or infinite solutions.

KEY TERMS

- break-even point
- system of linear equations
- substitution method
- consistent systems
- inconsistent systems

In today's world, the field of robotics is growing rapidly; however the interest and fascination with robots has been occurring for centuries. The word *robot* was first used in a 1920 play describing a factory that creates artificial people which could be mistaken for humans. However, the idea of *robots* did not begin there.

Descriptions of some of the first automatons, or self-operating machines, were recorded as early as the 3rd century BC! An ancient Chinese text describes a mechanical engineer presenting King Mu of Zhou with a life-size, human-shaped figure that could walk, move its head, and sing. By the 1200s, a Muslim engineer named al-Jazari created some of the first human-like machines that could be used for practical purposes. He created a drink-serving waitress and a hand-washing automaton that were both functional using hydropower. It is amazing to think of these people inventing such incredible robots without the use of today's technology.

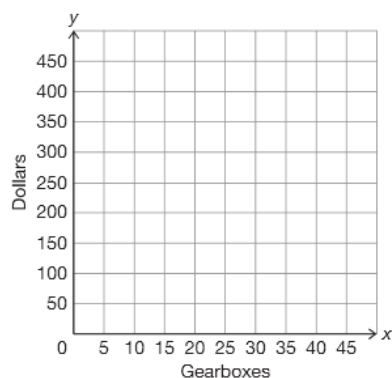
Are you surprised to learn that people created robots so long ago? What do we use robots for today? What do you think robots will be able to do in the future?

PROBLEM 1 Gearing For Success

Gwen has a part-time job working at Reliable Robots (RR) which sells electronics and hardware parts for robot creators. One of her tasks is to analyze RR's finances in terms of cost and income. Her boss, Mr. Robo, asks her to determine the *break-even point* for the cost and the income. The **break-even point** is the point when the cost and the income are equal. Gwen begins with the income and costs for gearboxes.



1. Let the function $I(g)$ represent the income (I) from selling gearboxes (g) and the function $C(g)$ represent the cost (C) of purchasing gearboxes (g).
 - a. Describe the relationship between the income function and the cost function that will show the break-even point. Explain your reasoning.
 - b. Describe the relationship between the income function and the cost function that will show a profit from selling gearboxes. Explain your reasoning.
2. RR purchases gearboxes from The Metalists for \$5.77 per gearbox plus a one-time credit check fee of \$45.00. RR sells each gearbox for \$8.50.
 - a. Write the function for the income generated from selling gearboxes.
 - b. Write the function for the cost of purchasing gearboxes from The Metalists.
3. Sketch a graph of each function on the coordinate plane to predict the break-even point of the income from RR selling the gearboxes and the cost of purchasing the gearboxes.



Be sure to label each graph so you know which graph represents cost and which represents income.



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a. How is the break-even point for $I(g)$ and $C(g)$ represented on the graph you sketched? Estimate the break-even point.

b. Could you determine the exact break-even point from the graph? Why or why not.

As you learned previously, the coordinates of an intersection point of two graphs can be exact or approximate depending on whether the intersection point is located on the intersection of two grid lines. You also learned that you had to use algebra to prove an exact intersection point.

Notice the units of measure for the independent and dependent quantities in $I(g)$ and $C(g)$ are the same.

In both functions $I(g)$ and $C(g)$, the g represents the independent quantity, gearboxes, and the dependent quantity is dollars. However, the dollars in each function represent different “types” of dollars. You know that dollars are represented differently because you determined two different functions—one function for cost (in dollars) and one function for income (in dollars).

When determining the break-even point algebraically between two functions, it is more efficient to transform each function into equation form. In this case, by transforming the functions into equation form, you establish one unit of measure for the dependent quantity: dollars.

Recall that in *A New Way to Write Something Familiar* in Chapter 1 Lesson 3, you transformed a function in equation form into function notation to more efficiently represent the independent and dependent quantities.



Analyze the functions representing cost and income from gearboxes for Reliable Robots.

$$I(g) = 8.5g \quad C(g) = 5.77g + 45$$



Since g is the independent variable, you can represent g as x in equation form.



$$I(x) = 8.5x \quad C(x) = 5.77x + 45$$



Since both $I(g)$ and $C(g)$ represent the dependent quantity in dollars, you can represent each using y as the variable.



$$y = 8.5x \quad y = 5.77x + 45$$




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4. Do you think it is possible to use other variables instead of x and y when transforming a function written in function notation to equation form?

When two or more equations define a relationship between quantities, they form a **system of linear equations**.

5. What is the relationship between the two equations in this problem situation?

Remember, a solution for a system of linear equations occurs when the values of the variables satisfy *all* of the linear equations.



Now that you have successfully created a system of linear equations, you can determine the break-even point for the gearboxes at RR. One way to solve a system of linear equations is called the *substitution method*. The **substitution method** is a process of solving a system of equations by substituting a variable in one equation with an equivalent expression.

Consider the system of equations from the previous worked example.

$$\begin{cases} y = 5.77x + 45 \\ y = 8.5x \end{cases}$$

Substitute the variable y in the first equation with the equivalent expression in the second equation.


$$8.5x = 5.77x + 45$$

Isolate the variable to solve.

$$2.73x = 45$$

$$x \approx 16.48$$

Since both equations are equal to y , we can set the equations equal to each other.

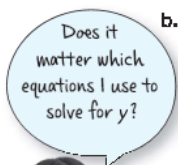


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6. Analyze the solution $x \approx 16.48$.

- a. What does this point represent in terms of the problem situation? Why is this solution an approximation?



b. Solve for y . Describe the solution in terms of this problem situation.



c. What is the profit from gearboxes at the break-even point?

d. Does this break-even point make sense in terms of the problem situation? Why or why not.

7. Analyze your graph of the cost and the income for the different number of gearboxes.

- a. Draw a box around the portion of the graph that represents when RR is losing money. Then write an inequality to represent this portion of the graph and describe what it means in terms of the problem situation.

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- b. Draw an oval around the portion of the graph that represents when RR is earning money. Then write an inequality to represent this portion of the graph and describe what it means in terms of the problem situation.



- c. Write an equation to represent the portion of the graph that represents when RR breaks even and describe what it means in terms of the problem situation.

PROBLEM 2 Saving Up



Marcus and Phillip are in the Robotics Club. They are both saving money to buy materials to build a new robot. They plan to save the same amount of money each week.

1. Write a function to represent the time it takes Marcus and Phillip to save money. Define your variables and explain why you chose those variables.

Marcus decides to open a new bank account. He deposits \$25 that he won in a robotics competition. He also plans on depositing \$10 a week that he earns from tutoring. Phillip decides he wants to keep his money in a sock drawer. He already has \$40 saved from mowing lawns over the summer. He plans to also save \$10 a week from his allowance.

2. Write a function to represent the information regarding Marcus and Phillip saving money for new robotics materials.
3. Predict when Marcus and Phillip will have the same amount of money saved. Use your functions to help you determine your prediction.

You can prove your prediction by solving and graphing a system of linear equations.

4. Rewrite each function as an equation. Use x and y for the variables of each function in equation form and define the variables. Then, write a system of linear equations.

5. Analyze each equation.
 - a. Describe what the slope of each line represents in this problem situation.

 - b. How do the slopes compare? Describe what this means in terms of this problem situation.

 - c. Describe what the y -intercept of each line represents in this problem situation.

 - d. How do the y -intercepts compare? Describe what this means in terms of this problem situation.

6. Determine the solution of the system of linear equations algebraically and graphically.
 - a. Use the substitution method to determine the intersection point.

 - b. Does your solution make sense? Describe what this means in terms of the problem situation.

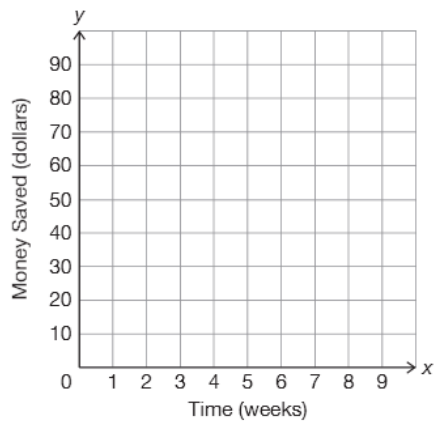
Can you have a solution without variables?



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c. Predict what the graph of this system will look like. Explain your reasoning.

d. Graph both equations on the coordinate plane provided.



7. Analyze the graph you created.

a. Describe the relationship between the graphs.

b. Does this linear system have a solution? Explain your reasoning.

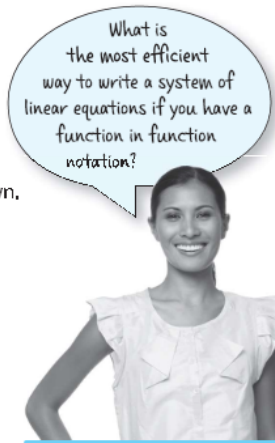


8. Was your prediction in Question 3 correct? Explain how you algebraically and graphically proved your prediction.

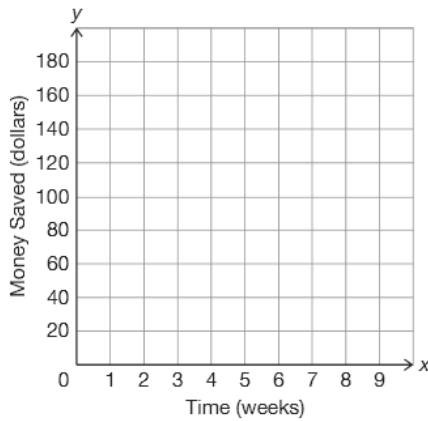


9. Tonya is also in the Robotics Club and has heard about Marcus's and Phillip's savings plans. She wants to be able to buy her new materials before Phillip, so she opens her own bank account. She is able to deposit \$40 in her account that she has saved from her job as a waitress. Each week she also deposits \$4 from her tips.
- Write a function that represents the information about Tonya saving money every week. Do not forget to define your variables.

- Write a linear system to represent the total amount of money Tonya and Phillip have after a certain amount of time.



- Graph the linear system on the coordinate plane shown.



10. Do the graphs intersect? If so, describe the meaning in terms of this problem situation.



11. Phillip and Tonya went on a shopping spree this weekend and spent all their savings except for \$40 each. Phillip is still saving \$10 a week from his allowance. Tonya now deposits her tips twice a week. On Tuesdays she deposits \$4 and on Saturdays she deposits \$6. Phillip claims he is still saving more each week than Tonya.
- Do you think Phillip's claim is true? Explain your reasoning.

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b. How can you prove your prediction?

12. Prove your prediction algebraically and graphically.

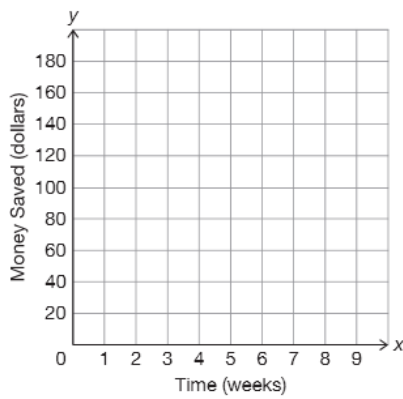
a. Write functions that represent any new information about the way Tonya and Phillip are now saving money.

b. Write a new linear system to represent the total amount of money each friend has after a certain amount of time.

When naming your functions, be sure you can tell the difference between these functions and the ones you previously wrote. If you used $P(w)$ to represent the amount of money that Phillip is saving each week, you might consider using subscripts like $P_2(w)$ to represent the new function for the money he is saving each week.



c. Graph the linear system on the coordinate plane.



13. Analyze the graph.

a. Describe the relationship between the graphs. What does this mean in terms of this problem situation?

b. Algebraically prove the relationship you stated in part (a).

- c. Does this solution prove the relationship? Explain your reasoning.



14. Was Phillip's claim that he is still saving more than Tonya a true statement? Explain why or why not.

PROBLEM 3 Transforming Equations: More Than Meets the Eye



Not all systems will be written in slope-intercept form or function notation. Systems can also be written in standard form. Let's explore a system in standard form.

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

Do you think there is more than one way to transform one of the equations in the system to create a new equation with only one unknown?

- Analyze each student's work.

👍 Dontrell

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

$$2x + 8y = 10$$

$$4x + 2 = y$$

$$2x + 8(4x + 2) = 10$$

👍 Janelle

👍 Maria

$$\begin{cases} 2x + 8y = 10 \\ 4x = y - 2 \end{cases}$$

$$8y = -2x + 10 \quad 4x = y - 2$$

$$y = \frac{2}{8}x + \frac{10}{8} \quad 4x + 2 = y$$

$$\frac{2}{8}x + \frac{10}{8} = 4x + 2$$

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- a. Describe the method Dontrell used to solve this system of equations and explain why he is correct.

b. Describe the method Janelle used to solve this system of equations and explain why her reasoning is correct.

c. Describe the method Maria used to solve this system of equations and explain why her reasoning is correct.



2. Which method do you prefer for solving this system of equations?



3. Use one of the methods shown or use your own method to determine the solution to this system of equations.

4. Soo Jin encountered this system of linear equations.

$$\begin{cases} 3.5x + 1.2y = 8 \\ 4.7x + 0.3y = 10.3 \end{cases}$$


However, Soo Jin has decimaphobia—a fear of decimals! Sammy tells her she has nothing to fear. He says, “All you need to do is multiply each equation by 10 to transform the system into whole numbers.”

a. Is Sammy correct? Explain why or why not.

Soo Jin!
There is no
such thing as
decimaphobia!



- b. Soo Jin attempts Sammy's method. Her work is shown.

 **Soo Jin**

$$\begin{cases} 35x + 12y = 8 \\ 47x + 3y = 103 \end{cases}$$



Explain the mistake(s) Soo Jin made and then determine the correct way to rewrite this system.

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Talk the Talk



1. Use any method of substitution to determine the solutions for each of the systems of linear equations.

a.
$$\begin{cases} 8x - 2y = 7 \\ 2x + y = 4 \end{cases}$$

b.
$$\begin{cases} 0.4x + 0.3y = 1 \\ 0.1y = 0.2x \end{cases}$$

c.
$$\begin{cases} \frac{1}{2}x + \frac{1}{4}y = 6 \\ y = 4 \end{cases}$$

d.
$$\begin{cases} 6x + 3y = 5 \\ y = -2x + 1 \end{cases}$$

A system of equations may have one unique solution, infinitely many solutions, or no solution. Systems that have one or many solutions are called **consistent systems**. Systems with no solution are called **inconsistent systems**.

2. Complete the table.

	Consistent Systems		Inconsistent Systems
Number of Solutions			
Description of y-intercepts	y-intercepts can be the same or different	y-intercepts are the same	y-intercepts are different
Description of Graph			

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Be prepared to share your solutions and methods.